Integrály z goniometrických funkcí

1) $\int \cos^3 x \sin^2 x \,\mathrm{d}x$ 2) $\int \sin^5 x \cos^4 x \,\mathrm{d}x$ 3) $\int \cos^5 x \,\mathrm{d}x$

4) $\int \sin^3 x \,\mathrm{d}x$ 5) $\int \cos^3 x \sin^3 x \,\mathrm{d}x$ 6) $\int \sin^3 x \cos^2 x \,\mathrm{d}x$

7) $\int \frac{1}{\cos x} \,\mathrm{d}x$ 8) $\int \frac{2}{5 - 3\cos x} \,\mathrm{d}x$ 9) $\int \cos^4 x \,\mathrm{d}x$

10) $\int \frac{1}{1 + \sin x} \,\mathrm{d}x$ 11) $\int \frac{1}{\cos^4 x} \,\mathrm{d}x$ 12) $\int \sin^2 x \cos^2 x \,\mathrm{d}x$

13) $\int \frac{\cos x}{\cos^2 x - \sin x + 1} \,\mathrm{d}x$ 14) $\int \frac{1 - \cos x}{(1 + \cos x)\sin x} \,\mathrm{d}x$ 15) $\int \mathrm{tg}^4 x \,\mathrm{d}x$

16) $\int \mathrm{cotg}^2 x \,\mathrm{d}x$ 17) $\int \frac{1 - \sin x}{1 + \cos x} \,\mathrm{d}x$ 18) $\int \frac{\mathrm{d}x}{1 - \cos x}$

Vzorce:

$$I(n) \overset{\text{ozn}}{=} \int \frac{\mathrm{d}x}{(1 + x^2)^n}$$

$$I(n) = \frac{1}{2n-2} \cdot \frac{x}{(1 + x^2)^{n-1}} + \frac{2n-3}{2n-2} \cdot I(n-1)$$

Výsledky

1) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c\,, \quad x \in R \quad (\text{substituce } t = \sin x)$

2) $-\frac{\cos^5 x}{5} + 2\frac{\cos^7 x}{7} - 9\frac{\cos^9 x}{9} + c\,, \quad x \in R \quad (\text{substituce } t = \cos x)$

3) $\sin x - 2\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + c\,, \quad x \in R \quad (\text{substituce } t = \sin x)$

4) $-\cos x + \frac{\cos^3 x}{3} + c\,, \quad x \in R \quad (\text{substituce } t = \cos x)$

5) $\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + c\,, \quad x \in R \quad (\text{substituce } t = \sin x, \text{ nebo } t = \cos x)$

6) $-\dfrac{\cos^3 x}{3}+\dfrac{\cos^5 x}{5}+c\,,\quad x\in R\quad(\text{substituce } t=\cos x)$

7) $\dfrac{1}{2}\ln\left|\dfrac{1+\sin x}{1-\sin x}\right|+c\,,\quad x\in R\setminus\left\{(2k+1)\dfrac{\pi}{2}\right\}\,,\ k\in Z\quad(\text{substituce } t=\sin x)$

8) $\operatorname{arctg}\left(2\operatorname{tg}\dfrac{x}{2}\right)+c\,,\quad x\in R\setminus\{(2k+1)\pi\}\,,\ k\in Z\quad\left(\text{substituce } t=\operatorname{tg}\dfrac{x}{2}\right)$

9) $\dfrac{3}{8}(x+\sin x\cos x)+\dfrac{1}{4}\sin x\cos^3 x+c\,,\quad x\in R$

10) $\operatorname{tg} x-\dfrac{1}{\cos x}+c\,,\quad x\in R\setminus\left\{(2k+1)\dfrac{\pi}{2}\right\}\,,\ k\in Z$

11) $\operatorname{tg} x+\dfrac{\operatorname{tg}^3 x}{3}+c\,,\quad x\in R\setminus\left\{(2k+1)\dfrac{\pi}{2}\right\}\,,\ k\in Z\quad(\text{substituce } t=\operatorname{tg} x)$

12) $\dfrac{1}{8}\left(x-\dfrac{\sin 4x}{4}\right)+c\,,\quad x\in R$

13) $\dfrac{1}{3}\ln\dfrac{2+\sin x}{1-\sin x}+c\,,\quad x\in\left(-\dfrac{3}{2}\pi+2k\pi,\,\dfrac{\pi}{2}+2k\pi\right)\,,\ k\in Z\quad(\text{substituce } t=\sin x)$

14) $\dfrac{1}{1+\cos x}+c\,,\quad x\in R\setminus\{(2k+1)\pi\}\,,\ k\in Z\quad(\text{substituce } t=\cos x)$

15) $\dfrac{1}{3}\operatorname{tg}^3 x-\operatorname{tg} x+x+c\,,\quad x\in R\setminus\left\{(2k+1)\dfrac{\pi}{2}\right\}\,,\ k\in Z\quad(\text{per partes})$

16) $-\operatorname{cotg} x-x+c\,,\quad x\in(0+k\pi,\,\pi+k\pi)\,,\ k\in Z$

17) $\operatorname{tg}\dfrac{x}{2}-\ln\left(1+\operatorname{tg}^2\dfrac{x}{2}\right)+c\,,\quad x\in(-\pi+2k\pi,\,\pi+2k\pi)\,,\ k\in Z\quad\left(\text{substituce } t=\operatorname{tg}\dfrac{x}{2}\right)$

18) $\operatorname{cotg} x-\dfrac{1}{\sin x}+c\,,\quad x\in R\setminus\{k\pi\}\,,\ k\in Z$